Studes on the Oiliness of Liquids. VI[1]. Measurements of the Kinetic Friction Coefficients by the Method of Sliding Velocity.

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The present paper describes the measurements of the kinetic friction coefficients by the sliding velocities of a slider on an inclined surface. It is generally accepted, as Amontons's rule, that the frictional force is independent of the contact area of two solids, and is simply proportional to the total pressure or load acting normally to the sliding surfaces. This rule is considered to be applicable to both kinetic and static friction so far as the condition of either dry friction or boundary lubrication is satisfied. In the case of static friction, the conditions of the measurement are relatively simple, while in the case of kinetic friction, it is rather complicated, because the friction is influenced, for instance, by the velocity of a body in motion and temperature rise caused by the friction, etc.

The published results on the influence of velocity on dry friction are rather conflicting. Coulomb pointed out that the velocity has almost no influence upon the dry friction between metals. On the other hand, Galton and Westinghouse,[2] and Smith[3] indicated that the friction decreases profoundly as the velocity increases. Jacob[4] determined the coefficients of kinetic and static frictions from the measurements of the critical angle of the frictional surface and concluded that on the contaminated surfaces the kinetic friction coefficient μ_k differs from the static friction coefficient μ_s considerably and the sliding motion is accelerated, while in the case of clean surfaces, μ_k is equal or nearly equal to μ_s and the motion is uniform.

The reports on the boundary lubrication are also complicated. Deeley,[5] working on boundary lubrication, found that μ_s was independent

(1) The first to fifth reports are published in the following: Sameshima, Kidokoro, and Akamatu, this Bulletin, **11** (1936), 659; Akamatu and Sameshima, *ibid.*, **11** (1936), 791; Sameshima and Miyake, *ibid.*, **12** (1937), 96; Sameshima and Tsubuku, *ibid.*, **12** (1937), 127; Akamatu, *ibid.*, **13** (1938), 127.

(2) Galton and Westinghouse, *Engineering*, **26** (1878), 153.

(3) Smith, Archibutt, and Deeley, "Lubrication and Lubricants," 843, (1927).

(4) Jacob, *Ann. Physik*, **38** (1912), 126.

(5) Deeley, *Proc. Phys. Soc.* (London), **32** (1920), 1s.

of the load and that at slow speeds, μ_k was practically equal to μ_s. On the other hand, Wilson and Barnard,[(6)] using Deeley's apparatus, observed that μ_k decreased with increasing speed, while Kimball,[(7)] Jenklin and Ewing[(8)] showed that at slow speeds μ_k exceeded μ_s and it increased with increasing speeds. Further, Jacob[(4)] indicated that when the frictional surfaces are separated by oily films, friction obeys the Coulomb's rule, the motion of a body is accelerated on the surface of the inclination exceeding a certain critical angle, and the value of μ_k is independent of the velocity. Recently, Beare and Bowden,[(9)] using the Deeley machine for measuring μ_k, further studied the problem of friction precisely and determined the constancy of μ_k over the wide range of velocity at various conditions of boundary lubrication.

Apparatus and Measurement. The apparatus employed in the present experiment has been constructed as shown in Fig. 1. In this figure, AB is a glass tube of 3 cm. in diameter and 60 cm. in length. A slider, shown in Fig. 2, has been made of a glass tube of 2 cm. in diameter,

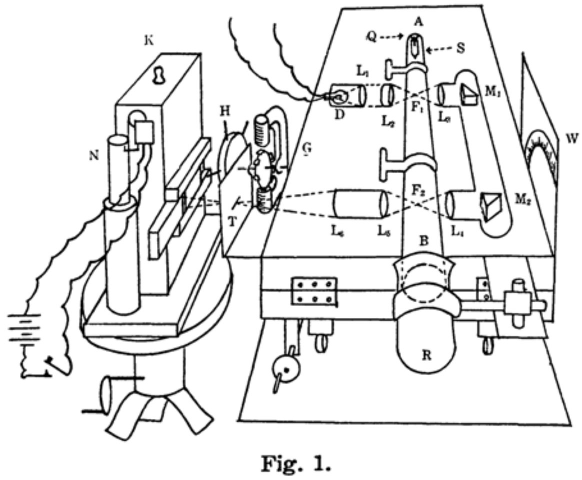

Fig. 1.

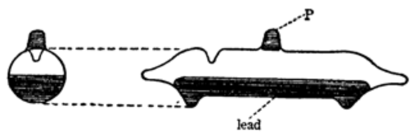

Fig. 2.

which is loaded with lead and carries a small projection P which serves to interrupts the optical paths in the tube AB. Again in Fig. 1, L_1, L_2, L_3, L_4, L_5, L_6, M_1, and M_2 are lenses and total reflexion prisms respectively. By this arrangement, the light from the source D is focussed

(6) Wilson and Barnard, *J. Ind. Eng. Chem.*, **14** (1922), 682.
(7) Kimball, *Am. J. Sci.*, [3], **13** (1877), 353.
(8) Jenklin and Ewing, *Phil. Trans.*, **167** (1877), 509.
(9) Beare and Bowden, *Phil. Trans.*, **234** (1935), 329.

at two points F_1 and F_2 in the tube AB and then, passing through the slit T, interrupted at constant time intervals by the projection H of a cynchronous motor G, it finally enters the time recorder box K, in which a photographic plate is made to slide down vertically, facing the light, with constant speed. The start of the plate is managed by the electromagnet N and a lever. The slider is held by the metal projection Q before its start and it rests, after each run, in the glass tube R containing cotton. The tube is made incline at a desired angle which is subsequently read by the protractor W.

The measurement is carried out as follows. At first the liquid under investigation is poured in the inclined tube, and the lever of the electromagnet N is pressed down to start the photographic plate. Then the slider is set in motion by releasing it from the support Q. Thus, the projection of the slider interrupts, as it runs down, at two points F_1 and F_2 of the optical path from the light source D to the photographic plate. On developing the plate, it gives the image of the line produced by D, having two dark spots caused by the interruption of the slider. At the same time, the cynchronous motor also marks a time scale on the line. Thus the time required for the passage between two points F_1 and F_2 can be measured to the accuracy of 1/50 second and therefore the mean velocity of the slider is calculated. The observations are undertaken at various inclinations of the tube.

In the measurement of the friction, the cleanliness of the sliding surfaces is important and it is necessary to remove the thin films of greasy matter on it. For this account, Hardy[10] paid special precautions for cleaning the surface. In the present experiment the following process has been adopted.[9] The tube is dipped in chromic acid mixture, washed carefully with soap solution, steaming with water vapour for 20 or 30 minutes, further with alcohol vapour for 20 or 30 minutes, and then finally dried by passing dried air through it. The slider is also cleaned by the same process prior to each experiment.

Calculation of the Kinetic Friction. In the present discussion it is assumed that the coefficient of the kinetic friction μ_K is constant and is independent of the sliding velocity, which has been verified in the present investigation. In Fig. 3 and in the following equations S_1 is distance between the starting point Q of the slider and F_1, S_2 distance between Q and F_2, T_1 time required to slide the distance S_1, T_2 time required to slide the distance S_2, m mass of the slider, θ angle of inclina-

(10) S. K. Hardy and W. B. Hardy, *Phil. Mag.*, [6], **38** (1919), 32.

tion of the tube, F driving force acting upon the slider along the tube, $v = 1/(T_2 - T_1)$ the quantity proportional to the mean velocity of the silder, G acceleration of the slider, and g gravity constant.

Then we obtain the following relations:

$$F = mg\sin\theta - \mu_k mg\cos\theta\ ,$$

$$G = \frac{F}{m} = g\sin\theta - \mu_k g\cos\theta \qquad (1),$$

$$S_1 = \frac{1}{2}GT_1^2 \qquad (2),$$

$$S_2 = \frac{1}{2}GT_2^2 \qquad (3),$$

$$v = \frac{1}{T_2 - T_1} \qquad (4).$$

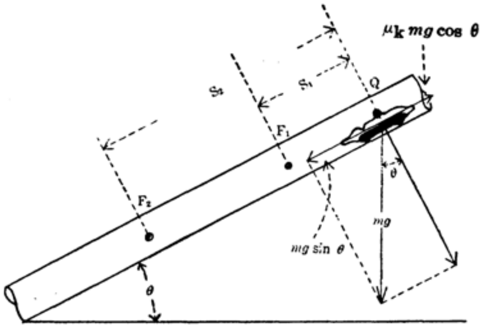

Fig. 3.

Eliminating T_1, T_2, and G from these four equations and expressing μ_k in terms of v and θ, we have the following equation:

$$\tan\theta - \frac{(\sqrt{2S_2} - \sqrt{2S_1})^2}{g} v^2 \sec\theta = \mu_k \qquad (5).$$

In this equation the value $(\sqrt{2S_2} - \sqrt{2S_1})^2/g$ is constant and so $\tan\theta$ is linear to $v^2 \sec\theta$. In the figure drawn by plotting $\tan\theta$ against $v^2 \sec\theta$, the point of intersection of the straight line with the axis of $\tan\theta$ gives the value of μ_k.

In order to test the above conclusion several liquids were used to construct the diagram of $\tan\theta$ and $v^2 \sec\theta$. Some typical examples are shown in the following tables and figures. Table 1 and Fig. 4 show the

Table 1. Glycerin.

θ	$\tan\theta$	T_2-T_1 (second)	$v^2 \sec\theta$	θ	$\tan\theta$	T_2-T_1 (second)	$v^2 \sec\theta$
40°	0.838	0.114	100.5	18°	0.324	0.184	31.0
38	0.780	0.112	101.0	16	0.286	0.210	23.5
36	0.726	0.118	88.9	15	0.268	0.216	22.2
34	0.675	0.120	84.8	14	0.249	0.232	20.0
32	0.625	0.124	76.5	13	0.231	0.246	17.0
30	0.575	0.130	68.5	12	0.212	0.278	13.2
28	0.531	0.140	57.9	11	0.194	0.298	11.5
26	0.487	0.140	56 9	10	0.176	0.380	7.0
24	0.445	0.144	52.8	9	0.158	0.420	5.7
22	0.403	0.150	47.9	8	0.140	0.470	4.6
20	0.364	0.166	38.5				

existence of the linear relation between $\tan\theta$ and $v^2 \sec\theta$ as given by equation (5). Most of the other liquids under examination showed the similar results. The angle of inclination of the straight line calculated from equation (5) gives 46°10′ on the scale of diagram employed in Fig. 4. The angles for many liquids range between 40° and 50°. Some liquids deviate from the linear relation, an example of which is shown in Fig. 5.

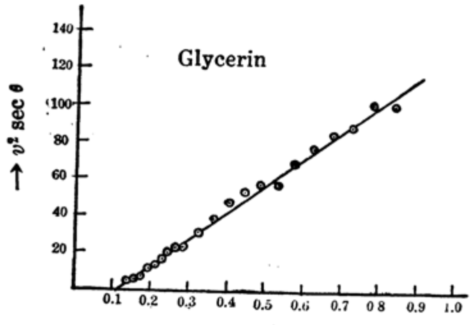

Fig. 4.

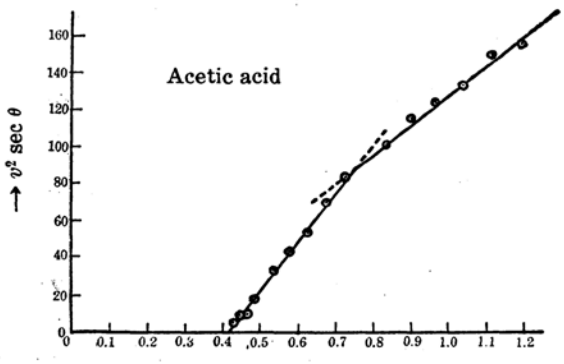

Fig. 5.

Examination of the Experimental Conditions. In order to test the reproducibility and to find the suitable experimental conditions, the following measurements were carried out on glycerin. The results are tabulated in Table 2, which gives the following conclusion. The reproducible value

Table 2.

Experimental conditions	μ_k (Glycerin)
(1) Using new tube of about 1 meter in length and new slider of 76.8 g. in weight.	0.10 0.10 0.09
(2) Using the above tube and slider after both of them have suffered abrasion by repeated experiments.	0.20 — —
(3) Using renewed tube of the same length as in (1) and renewed slider of 60.3 g. in weight.	0.10 — —

Table 2.—(*Concluded*)

Experimental conditions	μ_k (Glycerin)
(4) Using the same renewed tube as in (3) and abrased slider as in (2).	0.10 — —
(5) Using another new tube of 60 cm. in length and the abrased slider.	0.09 0.10 0.11

of μ_k can always be obtained with new tube and new slider, and it is independent of the length of the tube and the weight of the slider ((1), (3), and (5) in Table 2.), as is expected from equation (5). The value of μ_k remains unaltered so far as the glass tube is not abrased. The abrasion of both glass tube and slider increases the value of μ_k. In the present experiment the glass tube was always renewed when the reproducibility of μ_k with standard glycerin failed to exist.

Further it is necessary to start the slider with the force just to overcome the static friction. The excessive force imparts initial velocity to the slider which results the apparent decrease in the value of μ_k as shown in Table 3. In the experiments, pushing forces were controlled by the elastic strength of suitable spring arrangement. Too much force gives even a negative value of μ_k.

Table 3.

Force to overcome the static friction	Apparent μ_k of glycerin
Strongest	−0.03
Strong	0.05
Weak	0.10

Measurement of μ_k. The observed values of μ_k for water, alcohols, hydrocarbons, and fatty acid are tabulated in Table 4.

In the above observations, glycerin sometimes showed convex curves towards $\tan\theta$ axis owing perhaps to its high viscosity. When we plot the values of μ_k for alcohols against the numbers of carbon atoms in their molecules, the gradual decrease of μ_k with increasing numbers of carbon atoms is observed as in Fig. 6. These results may be compared with those of Beare and Bowden[9] or Sameshima and Miyake[1]. The μ_k value of water is large as is expected from the μ_s value of water.[1] The μ_k values

Table 4.

Lubricant	μ_k	Lubricant	μ_k
Glycerin	0.10	Hexane	0.34
Methyl alcohol	0.41	Heptane	0.16
Ethyl alcohol	0.34 (0.30)	Octane	0.24
Propyl alcohol	0.26	Nonane	0.31 (0.27)
Butyl alcohol	0.26 (0.23)	Acetic acid	0.42
Hexyl alcohol	0.15	Water	0.66 (0.69)

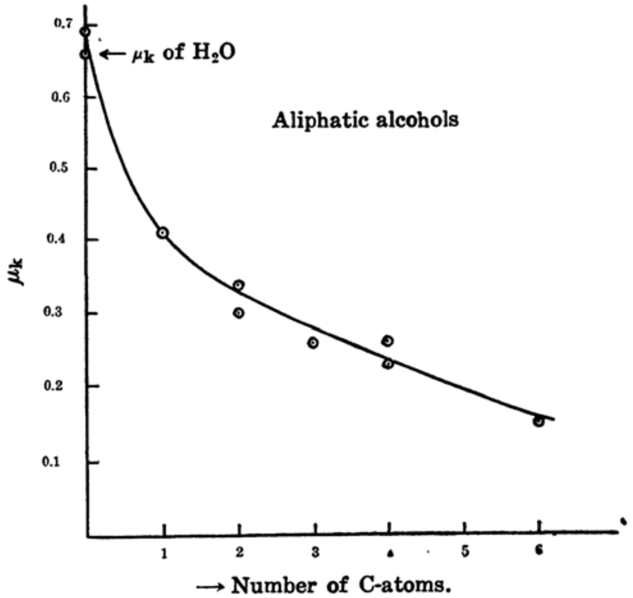

Fig. 6.

of hydrocarbons are irregular with respect to the numbers of carbon atoms in their molecules. Of the μ_k values of fatty acids series, only that of acetic acid was measured.

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Summary.

(1) Measurements of the kinetic friction coefficient have been made to study the oiliness of liquids. The method consists of the observation of the sliding velocity of a slider on an inclined surface lubricated with the liquid to be tested. Special apparatus has been constructed to measure the sliding velocity.

(2) It is confirmed that the kinetic friction coefficient is generally independent of the velocity over the range of the experiment (up to about 100 cm./sec.), and the following equation holds.

$$\tan\theta - \frac{(\sqrt{2S_2}-\sqrt{2S_1})^2}{g} v^2 \sec\theta = \mu_k$$

where v is the mean velocity of the slider on the surface of the inclination θ. The above equation lacks the viscosity term, so the conditons of the boundary lubrication are considered to be satisfied.

(3) The kinetic friction coefficients of glycerin, aliphatic hydrocarbons, aliphatic alcohols, acetic acid, and water have been measured.

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